"Manipulating Maps and Winning Elections" is a challenging paper to comment upon. In part, this is because it is, in sum, convincing, and in part because it is merely the tip of Professor Johnston's iceberg: It rests upon (and cites) a score of his previous works, which have been produced over more than twenty-five years of scholarly activity. In particular, this paper brings to bear data from Johnston's unique and detailed research into the mechanics and effects of the UK's system for defining parliamentary constituencies (Rossiter, Johnston, & Pattie 1999), and analytical methods that he has invented or refined (Rossiter, Johnston, & Pattie 1997; Johnston & Pattie, 2000).

I do not intend to comment upon every aspect of this article in these remarks. Instead, I will attempt to place that work in a more general analytic framework, and to suggest several methodological refinements and alternative approaches. Since Johnston explicitly draws on the general themes of "generalization and order," "measurement," (p. 2) and "understanding of aggregation patterns," (p. 3) and since he argues that significant insights are to be gained from the "application of mathematical and statistical reasoning," (p. 19) it is appropriate to put Johnston's approach into a more formal statistical framework. In the next section of this paper I present...
one such framework. In the subsequent sections, I then trace the implications that this framework suggests. I hope that this is useful both for those scholars who wish to apply the methods Johnston describe, but lack his extensive practical knowledge, and for more experienced scholars who wish to extend or supplement these methods.

A statistical framework for the evaluation of gerrymandering claims

Johnston defines gerrymandering in a way that is consonant with ordinary usage: “Gerrymandering involves careful drawing of constituency boundaries by a party so that either it wins a particular seat or, more generally, it wins more seats than its opponents.” (p. 5, emphasis added) And one of the fundamental questions that Johnston wishes to answer is: “Have [such] gerrymanders advantaged one party over another, and to what extent?” 2 (p. 2) Or, to put the question (approximately) contra-

other scholars to carry out similar analyses on other data sets.” Some empirical journals, like Political Analysis, are even more specific, requiring that every empirical paper specify an archive where the data used in the paper and the computer code necessary to reproduce the results in the paper, can be obtained.

That more attention to replicability would be useful is clearly demonstrated, when such a solidly empirical paper as this, by an author who has been in the forefront of making geographic data available for research (Johnston, Dorling, & Pattie, 1999), still falls short of full replicability. For example, the replicability of Johnson’s analysis would be improved by citation of the particular data sources used in the analysis, including any relevant information regarding versions, corrections, and recodings, in sufficient detail that another scholar could independently re-create or retrieve (from a data archive) that data and duplicate the analysis.

One should not take this parenthetical call for replication standards as a criticism of Johnston in particular. Quite the opposite, Johnston exceeds the norm in the level of care he takes to make his research reproducible and is to be applauded for it. The lesson to take from this, instead, is that making research articles replicable requires particular attention and systematic methods.

2 Johnston also prefaces this question with: “we have plenty of appreciation of particular cases. But that is all they are particular cases. We know a lot about gerrymandering of various types in the USA over recent decades… But we lack any further appreciation of how widespread such practices are, and what their impact on election results has been.”

I agree that the interaction between constituency boundaries and electoral results, especially at the national level, deserves more study. Scholars who are considering contributions in this area should however, be cognizant of relatively recent studies that do go beyond the examination of particular cases, and that assess the impact and pervasiveness of gerrymandering, and its impact on national elections. For example:

• Cox and Katz (1999, 2002) analyze elections to the U.S house across several decades, and argue that the changes following Baker v. Carr help explain both the persistence of the Democratic majority and the size of incumbency advantage.
• Part of my own dissertation research (Altman, 1998) examines the relationship between ‘traditional’ geographic indicia of gerrymanders, such as non-compact/ugly/bizarre districts and the electoral characteristics of those plans, such as responsiveness and bias. This is a national study, and covers the period from 1960–1990.
• Gelman and King (1994a) analyze elections in all state legislatures elected (exclusively) from single-member districts over the period 1968–1988, and conclude that redistricting increases electoral responsiveness, and also leads to less bias than would be expected if no redistricting had occurred. (Oddly, although Johnston does not include this result as an important contribution to our knowledge that goes beyond individual cases, he later cites Gelman and King as ‘important.’) Gelman and King (1994b)
positively, what would the electoral results have been had parties not attempted to influence the districting process? We cannot adequately answer this question without an analysis of counterfactuals, in other words, without (implicitly or explicitly) comparing alternative hypotheses explaining these election results and the predictions generated by these hypotheses.3

How do we formalize this evaluation? For simplicity, let us consider only the problem of detecting direct manipulation of district lines to achieve a desired outcome. Stated more formally, to decide whether a plan is the result of intentional manipulation, we need to compare the probabilities of the observed characteristics of the plan in question occurring conditioned on various hypotheses about the intent of the planners. This section lays out a simple formal framework for analyzing these counterfactuals:

1. Let \( p \) refer to a specific district plan. A plan is a partition4 of the set of all population units blocks into a set of districts: \( p = \{d_1, \ldots, d_n\} \) (Where each district, \( d_i \), is a set of census blocks: \( d_i = \{x_{i1}, x_{i2}, \ldots, x_{in}\} \).

2. If the planner has direct control over the district, the planner chooses some plan \( p^* \) from the set of all possible (and legal) plans conditioned on their intent \( y \). (More generally, the planner makes some choice that affects \( \text{prob}(C, \tilde{E}) \) conditioned on \( y \).) For example, we might hypothesize, in a particular political context, that \( y \) could take on two values5: (0) intent only to pay attention to traditional

make similar, although abbreviated, substantive claims based on their analysis of elections in the U.S. House from 1900–1990.

• Niemi and Winsky (1992) examine the extent to which redistricting contributed to partisan bias in national elections in the 70’s and 80’s, and the relationship between bias and partisan control of the redistricting process.

3 Kousser expresses this point eloquently in his writings about evaluating the evidence for racial gerrymanders:

“When historians attempt to explain some event, they implicitly or explicitly choose between two or more possible explanations on the basis of the extant evidence, relevant theory, and analogies. To say that racial or sex discrimination motivated an action is to say that discrimination caused the action in some sense and that other possible rationales did not cause it, or were less important, or at least do not wholly exclude invidious discrimination as the cause. Explanation cannot be assessed independently, but only in relation other explanation.” (Kousser, 1991, 714)

4 A partition divides a set into component groups that are exhaustive and exclusive. More formally:

For any set \( X = \{x_1, x_2, \ldots, x_n\} \), a partition is defined as a set of sets \( Y = \{y_1, y_2, \ldots, y_l\} \) s.t.

(1) \( \forall x_i \in X \exists y_j \in Y \ s.t. \ x_i \in y_j \)

(2) \( \forall i, \forall j \neq i, y_i \cap y_j = \emptyset \)

5 I select these two hypotheses for simplicity of exposition, not because these are the only plausible hypotheses extant. The framework presented may be easily generalized to multiple hypotheses.
quantitative and organic districting principles, or (1) predominant intent to create a partisan gerrymander.

3. We observe some characteristics of the plan, and some characteristics of elections held within the plan: \( \{C, \hat{E}\} \) The latter, inevitably, includes some unobserved random component.\(^6\) So, while we may observe later the realization of the election, the planner could, at the time the district was created, calculate only the expected value (or, generally, posterior distribution) of the election results from that district.\(^7\) For example, \( C \) might include the population, partisan registration, and racial composition of each district, while \( \hat{E} \) might include the number of votes for each party in a particular election, the number of seats won, the number of ‘wasted’ votes for each party, and anything else that is calculable only after the election has occurred.

When is there strong evidence that the observed outcome was the likely result of a gerrymander? Only where:

\[
B = \frac{\text{prob}(\{C, \hat{E}\} | y = 1)}{\text{prob}(\{C, \hat{E}\} | y = 0)} = \frac{\text{prob}(y = 1 | \{C, \hat{E}\})/\text{prob}(y = 0 | \{C, \hat{E}\}) > 1}{\text{prob}(y = 1)/\text{prob}(y = 0)}
\]

The quantity \( B \) is known as the ‘Bayes Factor’ and reflects the weight of evidence for one model versus the other.\(^8\)

Usually, we will also be interested in how much gerrymandering affects political outcomes. There is no single way to calculate this, but a simple approach is: \( EV(\{C, \hat{E}\} | y = 1) - EV(\{C, \hat{E}\} | y = 0) \). We might use this approach, for example to answer the following question: How many seats would have been won, had no attempt to manipulate district lines been made?

To my knowledge, no one has previously made explicit this framework.\(^9\) In order to perform any estimation using this framework, one must posit specific statistical models for \( \text{prob}(\{C, \hat{E}\} | y = i) \) (e.g., see Cox & Katz, 1999; Cirincione, Darling, &

\(^6\) Note that the resulting gerrymander is not necessarily assumed to be effective in its intent. For example, a particular plan may be designed to give one party an advantage, but an unanticipated swing in the normal vote could render the results innocuous.

\(^7\) I will assume that we can estimate its expected value as well, although this may not be straightforward.

\(^8\) Use of the Bayes factor dominates Bayesian model testing. (See Gill 2002 for an excellent survey of Bayesian methods in the social sciences). Note that the likelihood ratio is a special case of the Bayes Factor where the prior probabilities of each model are assumed to be equal, and the models nest by sharing the same parameter space. Although no specific threshold values are required in Bayesian model testing, \( B > 100 \) has been suggested as decisive.

\(^9\) Parts of this approach, however, have been implicit in previous analyses of redistricting, such Kousser (1991) and Gronke and Wilson (1999), Cox and Katz (1999).
O’Rourke, 2000), which I will not attempt here.\(^{10}\) We can still, however, draw a number of general conclusions from the formal framework.

First, the characteristics of the plan or election results, \((C, \hat{E})\), no matter how detailed or intricately derived, are not enough, by themselves, to support a conclusion about manipulation, or its extent.

Second, we should be careful to separate observations of electoral characteristics, such as wasted votes and turnout, from the statistical expected values (or more generally, posterior distributions) of these characteristics. Some variance in elections may be a result of random variables, or variables out of the control of, or unknown to the district planner.

Third, we should be careful to account for all reasonable hypotheses about the intent of the district planners, and about what these other plausible hypotheses would imply for the observed characteristics of the plans.

“Brookes’ method” and ‘gerrymandering’

The general cautions above also apply to Brookes’\(^{11}\) method in particular. Brookes’ method is an algebraic formula for decomposing ‘bias’ (defined as asymmetry in the seats/votes response function).\(^{12}\) It is not in itself a measure of the effect of gerrymandering, nor a test for its presence.

Application of Brookes’ method of decomposition requires two major assumptions, the first of which Johnston acknowledges. First, Brookes’ formula applies only when each of the major parties receives the same share of the votes. And, since this is rarely witnessed in practice, Brookes method is usually applied to hypothetical election results, under the assumption that shifts in support among major parties take the form of an absolutely uniform percentage shift in every constituency. Second, it assumes that the relation among major party votes, minor party votes, turnout and seats won is entirely deterministic—implying no measures of confidence (or, equivalently, extreme overconfidence—a standard error of zero). Both of these assumptions are obviously false.

Still, all models in social science are false to some degree, and these assumptions could be justified as approximations. What is of deeper concern is not their falsity, \textit{per se}, but their testability. When we observe the partisan balance shifting in elections across time (and between redistrictings), how much uniformity across constituencies can we observe? How much would deviations from uniformity affect our

\(^{10}\) A number of procedures (including one by Johnston) have been proposed, however, for estimating (essentially) \(prob(\{C, \hat{E}\}|y = 0)\) (e.g., see Rossiter & Johnston, 1981; Cirincione et al., 2000). Although, I have argued, elsewhere (Altman & McDonald, 2000), that many of these methods have significant flaws, or exclude relevant alternative hypotheses.

\(^{11}\) In my discussion of Brookes’ method, all comments should be taken to refer specifically to Johnston’s, Rossiter’s and Pattie’s modification of it, as specified in Appendix A of (Johnston, Rossiter, & Pattie, 1999)

\(^{12}\) One might say that Brookes’ method reapportions bias. (Pun intended.)
estimates of bias and the inferences we make from them? There is often some information in the voting data itself that can be utilized to answer these questions. And, in fact, there is some evidence that both estimation variability (the variability that results from the limited number of observations we are able to collect) and fundamental variability (the variability that results from non-deterministic processes or unobserved variables in the real world) is significant in British multiparty elections (Katz & King, 1999). Although the analysis of multiparty elections with significant malapportionment raises methodological issues not present in the analysis of US elections, there still exist a number of statistical approaches to analyzing bias and ‘responsiveness’ in the multi-party context. These range from unsophisticated (and methodologically flawed) application of linear regression to seemingly unrelated regression, full-information maximum-likelihood, and multiple imputation approaches (Monroe, 1998; Katz & King, 1999; Honaker, Katz and King, 2001).

In addition Brookes’ original assumption of a uniform shift of votes in every constituency seems to be called into question by Johnston’s finding of substantial amounts of tactical voting. Since tactical voting occurs in response to constituency-level political opportunities, 13 which do not obtain in every constituency, it is unlikely that shifts in partisan support will be uniform across constituencies.

Assuming, however, for the sake of discussion, that the assumptions of determinism and uniformity hold approximately, it is still vital to understand the limits of what the ‘gerrymandering’ component of Brookes’ method is actually capturing. In, Johnston’s words (pg 9), this component captures “the impact [on bias] of the pattern and volume of surplus and wasted votes.” One might also say, with equal accuracy, that it captures all of the residual ‘bias’ 14 that is left after (hypothetically) removing (various forms of) malapportionment.

In particular, this ‘gerrymandering’ component could be a result of intentional manipulation of district lines, or of unintended consequences of the application of neutral districting principles where parties have differing ‘geographies of support’ (p. 10). On the other hand, portions of the other ‘malapportionment’ components might also be a result of gerrymandering (or, of attention to ‘organic’ criteria, etc.). In other words, the ‘gerrymandering’ component of Brookes’ method does not necessarily show gerrymandering, as it is normally understood (and as Johnston has previously defined it). 15

Moreover, an analysis of bias, as measured through Brookes’ or other methods, may miss other quantitative evidence of gerrymandering. It is neither a necessary nor a sufficient condition for gerrymandering. For example, a biased plan can result from a planner, blind to partisan consequences, who wishes to maximize the number

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13 In particular, Johnston posits that tactical voting is more likely to occur only in constituencies that have marginal seats.

14 In this section, I use ‘bias’ in the technical sense used in redistricting. Formally, bias is asymmetry in the seats/votes response function.

15 Johnston defines gerrymandering as intentional on page 5, and seems to use it in this sense frequently at the beginning of the paper (pages 2, 5, 6, and 7) and in the final (in text) usage quoting Rush.
of competitive districts. Conversely, if the expected normal vote is far from 50%, a planner intent on maximizing seats for their party could create a plan that is formally unbiased (i.e. the seats/vote response curve is perfectly symmetric), but which yields all seats to their party, under the expected normal vote. (Kousser, 1996) Furthermore, for two party systems, it has been shown formally that, the optimal partisan gerrymander becomes less biased and more responsive as the risk aversion of the party increases (Owen & Grofman, 1988), or as the partisan loyalty of constituents decrease (McDonald, 1999).

Johnston, recognizes this issue, at least in part, when distinguishing between ‘infrastructural’ gerrymanders and ‘gerrymander-by-persuasion.’ It is not, however, clear why the estimation of the effect of gerrymander-by-persuasion should be associated particularly with the ‘gerrymandering’ component of Brookes’ method. Nor is it clear what competing hypotheses for the bias reported by Brookes’ method are ruled out. Still, much of the actual evaluation of gerrymander-by-persuasion’s effectiveness comes close in spirit to the formal framework that I describe: To estimate the number of seats attributable to the gerrymander-by-persuasion strategy in the third review, Rossiter, Johnston, and Pattie (1999) compare the observed election results to estimates of what the election results would have been, had they been held in the pre-redistricting seats. (p. 350) For the fourth review, they also compare these results to estimates of what they would have been under the seats initially suggested by the boundary review commission, and under the plans suggested by each party. (p. 359) The first comparison does not capture completely the effects of ‘gerrymander-by-persuasion’ because it does not answer the question of what would have happened if the parties had not attempted to influence the process: How likely is it that the old seats would simply have been reused in this (counterfactual) case? The second set of comparisons seems to be moving in the direction of the framework that I describe: The initial recommendations of the commission seem like a rough approximation (neglecting fundamental and estimation variability) of what might be expected under an uninfluenced process, or $EV(\{C,\tilde{E}\}|y=0)$.

**Conclusion: from electoral asymmetry to second-order bias**

Methodological details notwithstanding, Johnston marshals convincing circumstantial evidence that many of the asymmetries in electoral results are intended results of electoral manipulation. One of the most fascinating aspects of Johnston’s analysis is how electoral plans, in addition to being subject to direct manipulation, through the drawing of district lines, can also be manipulated indirectly, through the rules and institutions that govern the creation of these plans. This is a critical facet of

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16 A biased plan could also result from a combination of geographic constraints on districting and differences in location and dispersion of support for each party. For example – if half the supporters of a party were concentrated in one corner of the country, while the others were dispersed perfectly evenly across the rest of the country, it might be impossible to create a geographically compact and unbiased plan.

17 See Rossiter, Johnston, and Pattie (1999) for a more detailed treatment of this.
redistricting, but one that most quantitative analyses elide.\textsuperscript{18} In particular, Johnston shows how political parties in the UK have attempted to gain or maintain an advantage by changing the timing of the redistricting process.

Neither the measurement of bias nor its further decomposition of bias is sufficient to demonstrate intentionality here: The analysis of the timing of changes in bias that is crucial in showing the role of intent in this case. As Johnston convincingly argues, both the timing of the redistricting and the timing of the electoral changes are telling: Labor is generally strongest in areas where populations are declining, this party gains by ‘creeping malapportionment,’ and hence attempts to delay the redistricting process. Conversely, the Conservatives attempt to call for redistricting early. Moreover, the dramatic post-reapportionment changes in bias, percent effective vote, and pattern of bias components shown in figures 8 and 9 of Johnston’s paper lends circumstantial weight to his argument.

Another example of indirect manipulation that Johnston exposes is ‘gerrymandering by persuasion.’ This strategy, which involved Labor’s mobilization of local politicians in each region to submit plans that, while tailored in terms of ‘organic’ redistricting principles, were to the party’s advantage. This mobilization enabled the Labor party to gain political advantage in the redistricting process, even where the boundary commissions that were drawing the district lines were largely apolitical.

Robert Nozick’s terminology\textsuperscript{19} is apt here. Nozick (1993) defines ‘first-level bias’ as the intent to apply standards or procedures in a discriminatory fashion, so as to advantage a particular group. In contrast, ‘second-level bias’ denotes the selection of standards, procedures, or institutions, which will applied without discrimination, but which are chosen to advantage a particular group. Systematic asymmetries in outcomes, such as differences in the ‘wasted vote’ between parties, can be directly evaluated through a straightforward application of quantitative tools. But these are not sufficient to demonstrate bias. To detect first- and second-level biases, quantitative analysis is necessary but not sufficient. These inferences require a more nuanced analysis that includes construction of the set of competing hypotheses that offer plausible explanations for the redistricting plan, examination of the political context of the redistricting, and evaluation of the circumstantial evidence for each proposed explanation of the observed asymmetry.

Although some questions remain unanswered, Johnston’s solidly empirical analysis offers important insights into redistricting. Particularly, it shows how the mechanisms of redistricting can be indirectly manipulated to take advantage of the electoral geography of support for partisan purposes.

\textsuperscript{18} A notable exception is Frank Parker’s discussion of racially motivated redistricting in Mississippi, which also shows how formally neutral principles, can be chosen in order to create a gerrymander. One example chronicled by Parker shows how the 1969 redistricting of Hines county used ‘neutral principles’ of equalization of population, land area, and county road mileage was used to ‘crack’ the urban black community. (Parker, 1990, p. 151–166)

\textsuperscript{19} Note that this use of ‘bias’ is distinct from the technical usage above.
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