Redistricting by Formula: An Ohio Reform Experiment

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Abstract

In the last decade, Ohio reformers advocated redistricting by formula: selecting the redistricting plan that scores best on a pre-defined objective scoring function that combines *prima facie* neutral criteria with political goals of plan fairness and district competition. In the post-2010 redistricting these reformers hosted a public competition where prizes were awarded to the best legal plan scored on the reformers' formula. The submitted plans provide a unique opportunity to evaluate how redistricting by formula may work in practice. Our analysis finds the public yields a broader range of redistricting plans, on indicia of legal and public policy interest, than developed by the state legislature. The Pareto frontier reveals plans that perform better than the legislature's adopted plan on one and two dimensions, as well as the reformers' overall scoring function. Our evaluation reveals minimal trade-offs among the components of the overall competition's scoring criteria, but we caution that the scoring formula may be sensitive to implementation choices among its components. Compared to the legislature's plan, the reform community can get more of the four criteria they value; importantly, without sacrificing the state's only African-American opportunity congressional district.

Redistricting is the process of redrawing electoral district boundaries. Ostensibly, those drawing the lines improve representation by achieving facially-neutral objective administrative criteria such as population equality, contiguity, and, depending on the state, other criteria such as compactness, county integrity, and maintenance of existing political and community boundaries, among others. However, redistricting authorities often have wide discretion to act within the bounds of these constraints, such that the administrative criteria may not significantly constrain the creation of districts to promote political goals. These political goals may include incumbent protection by drawing of uncompetitive, safe districts; racial gerrymandering designed to diminish the representation of a racial or ethnic group; and the maximization of expected seats won by a favored party.

A potential approach to prevent redistricting authorities from using the process to further their political goals is to explicitly incorporate and balance the administrative criteria against excessive political goals in a quantitative formula. Redistricting by formula is popular among some advocates, including scholars (Vickrey 1961), pundits, and politicians, who wish to remove politics from redistricting. In the words of then-governor Ronald Reagan, "There is only one way to do reapportionment—feed into the computer all the factors except political registration."[1] Yet, scholars (Nagel 1965; Altman and McDonald 2010) have cautioned the promise of computing and formulaic approaches to redistricting are more limited than what advocates may envision. Ohio reformers have been recent strong advocates for redistricting by formula. Twice, in 2009 and 2011, a coalition of good-government groups held redistricting competitions that awarded prizes to redistricting plans drawn by members of the public that scored best on a formula comprising well-

[1] Tom Goff, "Governor Urges Redistricting Plan Without Partisan Politics," *Los Angeles Times*, Jan 21, 1972, p. A3.

defined administrative and political criteria. We evaluate the plans produced by this approach, and compare them to the legislature's adopted plan. In addition to assessing how well this formulaic approach works in Ohio, particularly with regard to public involvement in plan development, we further illuminate scholarly debate regarding the trade-offs among various redistricting criteria.

Redistricting in Ohio

Ohio uses a dual method of redistricting state legislative and congressional districts. The Ohio Apportionment Board established in 1851 is among the oldest redistricting commissions used in the United States (Barber 1981). The commission has five members, three of whom are statewide elected officials are two are appointed by the state legislative leaders of the two major political parties. Until recently, the Apportionment Board drew state legislative districts under an elaborate set of criteria that, after respecting federal equal population requirements and voting rights protections, balanced population equality, compactness, and the splitting local political boundaries down to local election wards (Ohio Constitution Article XI § 3). In contrast, Ohio's constitution and statutes do not mention congressional redistricting, leaving that process to the state legislature, constrained only by federal requirements.

During the post-2010 census redistricting, Republicans held all three statewide elected offices, and thus held a 4-1 majority on the Apportionment Board. Following the Board's adoption of a legislative redistricting plan, Democrats alleged the Board placed partisan interests over state constitutional requirements. The Ohio Supreme Court ruled 4-3 in *Wilson* v. *Kasich* that, "As long as the 2011 apportionment plan satisfied the constitutional requirements set forth in Article XI, respondents were not precluded from considering political factors in drafting it" (p.7). The majority ruled that plaintiffs could not establish beyond a reasonable doubt that the Board had not given due deference to the Ohio constitution in adopting the redistricting plan, citing similar

findings in cases in Arizona and West Virginia in the previous decade. The dissenters in the case argued, "...the majority opinion erects a nearly insurmountable barrier to a successful constitutional challenge by assigning to the board's actions a blanket presumption of constitutionality and requiring proof beyond a reasonable doubt to establish that the plan fails to meet all constitutional requirements" (p. 23).

Government reform advocates are motivated by the deference courts often give to redistricting authorities to seek ways to constrain their ability to gerrymander. An Ohio reform option is redistricting by formula. Reformers qualified a 2005 Ohio ballot initiative that would, among other changes, amend the state constitution to shift congressional redistricting to the Apportionment Board and require it to adopt the legislative and congressional redistricting plan that scored best on an explicit formula balancing political and administrative goals.[2] The ballot initiative failed (Tolbert, Smith and Green 2009), but reformers continued their advocacy efforts by hosting congressional redistricting competitions in 2009 and again in 2011 that awarded prizes to the redistricting plan that a formula scored best. These efforts culminated in a 2012 ballot initiative, this time amending the process to create a "citizen" commission that would immediately redraw districts.[3] This attempt also failed, however, prominent elected officials pledged to support reform. In 2015 the legislature passed a bipartisan referendum that amended the state constitution's process and criteria for state legislative redistricting.[4] This effort was successful. Although it does not explicitly require a formula, the Ohio constitution now requires that the "statewide proportion of districts...shall correspond closely to the statewide preferences of the voters of Ohio" (Ohio

[2] See: http://www.sos.state.oh.us/sos/upload/elections/2005/OIR2005.pdf, accessed March 19, 2016.
[3] See: http://www.sos.state.oh.us/sos/upload/ballotboard/2012/2012stateissues.pdf, accessed March 19, 2016.
[4] See: http://www.sos.state.oh.us/sos/upload/ballotboard/2015/2015IssuesReport.pdf, accessed March 19, 2016.

Constitution Article XI § 3). Reformers have vowed to "...extend them (reform) to congressional districts, which are even more gerrymandered."[5]

Redistricting by Formula

Redistricting by formula is an old concept. Prior to the U.S. Supreme Court rulings in the 1960s requiring equal population districts, redistricting was often, in practice, synonymous with apportionment, a formula for assigning districts to political subunits -- typically state legislative districts to counties according to their population. Apportionment formulas were well-known to affect representation; for example, in 1776 Thomas Jefferson unsuccessfully proposed an apportionment formula for Virginia's legislature designed to shift political power to western regions (McKay 1965: 19). By the 1960s, where apportionment formulas assigned a minimal number of seats to a single political unit, severe population imbalances between rural and urban localities resulted in malapportionment enabling the formation of policies favoring rural over urban interests (Ansolabehere, Gerber, and Snyder 2002).

Chief Justice Earl Warren (1977: 306) called the equal population rulings of the 1960s the most significant of his career due to the transference of political power from rural to urban areas, where many minority communities where situated. At the time, some scholars believed that the equal population rulings would prevent future gerrymandering (White and Thomas 1964). Furthermore, in theory, adding a constraint such as equal population both makes gerrymandering more difficult computationally (Altman 1997; Puppe 2009) and reduces the expected advantage achievable through optimal gerrymandering (Sherstyuk 1998).

[5] See: Jim Segal, "Voters approve issue to reform Ohio's redistricting process," *The Columbus Dispatch*, Nov. 3, 2015, http://www.dispatch.com/content/stories/public/2015/election/ohio-state-issue-1-redistricting.html, accessed March 19, 2016.

The political reality of redistricting is more complex: in part because spatial, demographic, and political characteristics interact; in part because these characteristics change over time; and in part because actors adapt to rules and learn how to work within them. A subsequent retrospective assessment is that politicians adapted to the new regime and "learned how to take advantage of the equal population requirement" (Niemi and Winsky 1992: 566). Another, more nuanced evaluation, is that constraining malapportionment rectified political inequalities arising from population imbalances alone, since "population equality guarantees almost no form of fairness beyond numerical equality of population" (Gelman and King 1994: 553; c.f., Cox and Katz 2002).

There are two lessons relevant to Ohio's experience with redistricting by formula. Firstly, a formula may have political effects that can be readily predicted by stakeholders, such that redistricting by formula is transformed into gerrymandering by formula. Secondly, elements of a formula may not be sufficiently constraining, such that a formula's implementation leaves considerable room to achieve other goals. Furthermore, because identifying the optimal redistricting plan is generally computationally infeasible, the degree of constraint may, in practice, depend on the operational details (heuristic algorithm, software implementation, and human exploration) used to find a plan that optimizes a formula.

Numerous observers have noted that application of a nominally neutral formula may lead to outcomes with predictable biases. Nozick (1994: 103) sums up these issues cogently and refers generally to such bias and self-interested preferences over rules as "second-order." Parker (1990) documents an example of second order bias in redistricting: In the early 1970's Hinds County Mississippi Supervisors enacted a racial gerrymander purely through formula — by selecting districts based on equalized road mileage and population, thereby inevitably fragmenting the dense urban African-American community. Justice Byron White also reflected on this type of implicit

bias in *Gaffney* v *Cummings* (1973: 754), reacting to calls for a formulaic application of administrative criteria such as compactness and respect for local political boundaries he stated, "This politically mindless approach may produce, whether intended or not, the most grossly gerrymandered results."

Generally, second-order bias may be conceptualized as embodying trade-offs in implementing different redistricting criteria. Maximizing a criterion might come at the expense of another, when applied within a relatively-fixed geographic, political, or demographic context. Scholars have asserted such trade-offs exist between pairs of criteria, such as drawing compact districts and creating a "fair" redistricting plan reflecting the overall partisanship of a state (Altman 1998; Chen and Rodden 2013), compactness and effective representation for minority communities (Barabas and Jerit 2004), respect for political boundaries and partisan fairness (Winburn 2008), partisan fairness and competitive districts with a balance of partisans within them (Niemi & Deegan 1978), minority representation and partisan fairness (Shotts 2001; Brace, et al. 1987), and minority representation and competitive districts (McDonald 2006a).

It is, of course, possible that a criterion is not binding and therefore does not impose a constraint on another criteria. For example, the equal population requirement provided for a more equitable division of political power between urban and rural areas but appears to have not strongly constrained partisan gerrymandering. There is little scholarly research on the inability of one criterion to constrain another, perhaps because such "null results" are difficult to publish. However, scholars reacting to others' assertions of the existence of trade-offs have noted how compactness may not greatly constrain partisan or racial gerrymanders (Altman 1998; Klarner 2007). Much of the litigation over redistricting criteria involves state constitutional and statutory criteria, which are more varied than the limited federal criteria of equal population and voting rights—encompassing

criteria such as compactness and respect for existing political and community boundaries, among others (Cain, Mac Donald and McDonald 2005). Litigation over state criteria is increasing as reformers are successful at imposing new regulations. However, in any given redistricting cycle only a handful of courts have overturned adopted redistricting plans due to state criteria violations. Nonetheless, the absence of court action does not provide strong evidence of a criterion's effectiveness, since in equilibrium a rationally-acting redistricting authority would meet legal requirements to avoid a court from invalidating their work.[6]

Criteria focusing on 'communities of interest' have similarly been insufficient to constrain strongly political manipulation. Although many states have general requirements to respect communities of interest (NCSL 2009), and, in theory, measures applied to communities of interest could incorporate general representational criteria, this has not happened in practice within the United States. In practice, the communities of interest criterion has either been ignored, or limited to protecting against districts that would divide concentrated homogenous ethnic or racial groups. While this can indirectly constrain political goals, it may constrain one party substantially more than another (Altman and McDonald 2014) and generally leaves ample room for political manipulation. Indeed, when communities of interest lack definitional foundation any boundary line can be argued to respect *post hoc* communities.

If neither administrative criteria nor communities of interest are sufficient to constrain political goals, then a solution may be to explicitly incorporate political goals into the redistricting criteria. Princeton professor Donald Stokes (1993), serving as the ninth member of New Jersey's

[6] One might wonder why any court action occurs if redistricting authorities always follow the law. What is and is not constitutionally permissible may not be well-defined when no litigation has occurred, when precedent may be re-interpreted, or when the legal facts may be specific to the circumstances of continually politically and geographically evolving populations. A redistricting plan often has significant electoral effects, so a politically motivated redistricting authority might push the bounds of what is constitutionally permissible to achieve further electoral advantage. When a redistricting authority draws a plan into such a gray area, aggrieved parties have an incentive to litigate an unfavorable outcome on even the sometimes-small chance a court will invalidate the plan.

otherwise politically-balanced New Jersey Legislative Apportionment Commission, was perhaps the first to formally implement explicit political criteria during an official U.S. redistricting. He evaluated the two major political parties' redistricting plans using administrative criteria with an explicit goal of creating a plan that was politically fair as defined by quantitative metrics developed in the social sciences (e.g., Tufte 1973).[7] Stokes, and those who followed him, induced the parties to propose redistricting plans meeting his objectives by stating he would cast his tie-breaking vote for the plan that scored best on his criteria.

The New Jersey experience cannot fully illuminate how redistricting by formula constrains a redistricting authority because only a few plans are created in the process, and these are strongly motivated by strategic advantage. Each party reveals only a limited number of plans that attempt to jointly maximize the formula's criteria and partisan advantage. This framework may favor the dominant party, as scholars (Butler 1963; Lijphart 2012) note a plan that is formally unbiased—yielding an even number of seats to each party in an evenly divided election—may provide that party with a greater number of favorable districts over a proportional allocation in elections where the party wins more than fifty percent of the vote.[8] As an illustration of this phenomenon, the tie-breaking members succeeding Stokes who have employed his method have voted with the Democrats in every redistricting cycle.

Ohio reformers wish to expand the scope of participation beyond the political parties by enabling public submissions of redistricting plans. As a proof of concept, reformers hosted two redistricting competitions, in 2009 and 2011, where internet-based software (Salling 2010; Geier

[7] Stokes' method averaged legislative election results within districts and computed a partisan bias measures in a simulated dead-heat "50-50" election, i.e., the percentage of seats won above or below fifty percent if a party hypothetically won fifty percent of the vote. Subsequent ninth members incorporated statewide elections into the computations.

[8] It is for this reason that some scholars (Grofman and King 2007) advocate for partisan symmetry, that both parties win the same number of seats for a given vote share for that party, over explicit proportional representation as a partisan gerrymandering standard.

2011) enabled the public to draw redistricting plans and awards were given to the congressional and state legislative plans that scored best on a defined formula composed of measures and a weighting scheme for the criteria of compactness, respecting county boundaries, partisan fairness, and competition (we elaborate below). This was not the only public participation effort during the 2010 redistricting cycle. Participants in a 2011 Virginia redistricting competition, and those using the Florida legislature's public-facing redistricting mapping application, approached redistricting differently than politicians (Altman and McDonald 2013, 2014)—exploring more of the potential tradeoffs, and producing plans that better achieved good-government criteria. The Virginia competition promoted similar criteria as Ohio's competition. However, the Virginia contest organizers only asked participants to achieve as best they could the judging criteria without explicitly maximizing a fixed scoring function. The explicit formula in Ohio's contest thus allows us to analyze data well-suited to evaluate scholarly claims about trade-offs among components of the scoring function, and as a result, to evaluate the degree to which redistricting by formula might be "gamed" for political gain.

Additional plans can help further delineate the feasible space of redistricting plans as they are scored on various criteria, but this exercise will always be incomplete. To return to the language of optimization algorithms, redistricting is a computationally complex task belonging to a special class of partitioning problems known to be NP-hard (Altman 1997). In even a small state, there are more ways to assign tens of thousands of census blocks to districts than there are quarks in the universe. Unlike well-behaved single-peaked functions for which a computer algorithm can quickly find the optimum, in most circumstances algorithms designed to find optimal solutions to NP-hard problems can become easily trapped in local optima and are not guaranteed to find the optimal solution in a reasonable amount of time—given current computing capabilities, such a

solution might be on the order of billions of years. Although scholars have claimed to have devised algorithms to sample the space of feasible redistricting plans using optimization algorithms (Cirincione, Darling and O'Rourke 2000; Chen and Rodden 2013), automated redistricting algorithms are likely to favor certain types of local optima over others, and because enumeration of all plans is infeasible, there is no way to definitively prove whether the potential biases of automated algorithms affect criteria of interest (see Altman and McDonald 2010: 98; Altman 1997).

Humans use a variety of different heuristic algorithms and employ outside knowledge. They are thus sometimes able to creatively see past local optima where a computer, running a fixed algorithm, might otherwise become trapped. However, engaging human computers in the task does not alleviate these fundamental mathematical problems—in consequence, while plans produced by either machine or humans can suggest potential trade-offs among criteria, the mapping of the frontiers among criteria will likely remain incomplete. Nonetheless, there remain benefits in the existence of more plans drawn from varying perspectives, since they can more fully illuminate the feasible spaces for redistricting plans, and may better inform us about potential trade-offs.

Data and Methods

Here, we analyze all publicly available and objectively legal Ohio congressional redistricting plans produced in the wake of the 2010 census. We obtained these plans from three sources: the legislature's published adopted plan, plans publicly shared on the advocates' competition website, and a plan produced by the DrawCongress project. The competition

organizers further required plans to meet mandatory criteria to qualify for judging; these plans were a subset of those shared on the competition's website.[9]

We evaluated Ohio congressional redistricting plans using two sets of criteria, those formulated for the competition and those that scholars have used to evaluate redistricting plans. These criteria sets are closely related. They encompass the same concepts of population equality, minority representation, respect for county boundaries, district compactness, the number of competitive districts, and the plan's partisan fairness. The scoring for these concepts are also similar, but different enough in some instances that we present parallel analyses to be consistent with scholarly research and with advocates' goals.

Congressional plans submitted to the 2011 Ohio redistricting competition were evaluated on two categories of criteria, mandatory (i.e., threshold) criteria and permissive scoring criteria. Mandatory criteria are those necessary to meet minimal requirements for a redistricting plan to be legal under federal and state law — these include completeness, contiguity, equal population and voting rights.[10] Scoring criteria are additions that the competition hosts decided were important values that they wished to encourage contestants to maximize. These scoring criteria included respect for county boundaries, compactness, partisan fairness, and competitive districts. The competition was intended as a vehicle to demonstrate the feasibility of formally adopting these criteria—reformers have twice incorporated these scoring criteria into ballot initiatives, only to

[9] The adopted congressional plan is available at: http://www.sos.state.oh.us/elections/candidates/District%20Maps.aspx. The Draw Congress map is available at: http://web.law.columbia.edu/redistricting. We obtained the competition plans from the competition website, http://drawthelineohio.org, before it became defunct. We provide all the the machine readable definitions of these plans, along with the scores calculated for the analysis in this paper, and the related demographic and electoral data are available at (Altman & McDonald 2017).

[10] The scoring rules are no longer available on the web. We obtained the scoring rules, which we cite and will provide as part of the replication file for this paper upon manuscript publication.

have them later rejected by Ohio voters. By fortunate coincidence these criteria are also of interest to scholars.

The mandatory criteria, which both we and the competition organizers agree a plan must meet to be legal, consist of the following: a plan must have sixteen congressional districts, the number allotted to Ohio following the 2010 census; it must be complete, in that all census blocks (the smallest geographic unit used in Ohio's redistricting) must be assigned to a district;[11] it must be contiguous, in that all census blocks must connect;[12] and the population of each district must be approximately equal.

There is some flexibility regarding the allowable deviation for population equality. Advocates required that plans scored in the competition have a population deviation between the largest and smallest district of no more than a range of one percent of the ideal average population district.[13] This is also a recognized legal requirement; however, in practice redistricting authorities generally prefer to enact plans with absolute minimal population deviations to seek a safe harbor against litigation.[14] Some contestants drew redistricting plans to this exacting requirement. All plans we collected, excepting some plans shared on the competition website, met the population equality threshold. We excluded plans failing to meet the threshold from our analysis.

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[11] Competition organizers excluded from their database census blocks located in Lake Erie. Thus, it was impossible for users to assign all 2010 census blocks to districts.

[12] Competition organizers required districts not to be "point contiguous" by connecting blocks only at their vertices. Competition organizers further simplified the Census Bureau's geographic data by modifying and consolidating water blocks. As a consequence, competition plans were incomplete, in that all census blocks as defined by the Census Bureau were not assigned to a district. We decided to analyze competition plans where unassigned census blocks had zero population, which may slightly affect their compactness scores since compactness measures are defined in terms of districts' geography.

[13] This population deviation arises from the deviations we observe among the competition plans. The competition rules formally state a threshold deviation of nor more than plus or minus 0.5% from the ideal average population district.

[14] Courts allow population deviations if a jurisdiction can show a compelling interest for them. For example, Iowa and West Virginia congressional districts during the 2000's had modest population deviations because districts did not split county boundaries.

Ohio, like the entire country, is governed by Section 2 of the Voting Rights Act, which requires the creation of majority-minority districts under specific circumstances.[15] The legislature's adopted plan has one majority African-American voting-age population congressional district located in Cuyahoga County, the only such district possible to draw in the state. There has been no Voting Rights Act Section 2 litigation to compel Ohio to draw a minority opportunity district, so it is unclear if Ohio is bound to draw this district and it may be possible to elect an African-American candidate in a district with less than a majority African-American voting-age population. Competition organizers required plans qualifying for scoring to have one African-American voting-age population (near) majority district, with a slightly lower African-American voting-age population of 48%.[16] All plans we collected, excepting some plans shared on the competition website, but not submitted for judging, had one (near) minority-majority congressional district.

Districts split localities when two or more districts are assigned to a locality's geography. We score the number of county splits in the following manner: When more than one district is located in a county, each district located in the county is scored as splitting the county. The competition organizers used a similar scoring rule, except that districts entirely contained within a county without crossing the county line, as is possible only in Cuyahoga County, do not count as a fragment. County fragments are then summed across the entire state, and for the competition

[15] The U.S. Supreme Court articulated the three-pronged "Gingles test" in *Thornburg* v *Gingles* 478 U.S. 30 (1986) which requires the creation of a minority-majority district if it is possible to draw such a district in a compact manner, if there is the presence of racially polarized voting, and given the totality of the circumstances. See also, *Bartlett* v *Strickland* 556 U.S. 1 (2009) where the Court explicitly required the minority community to be a majority of a demonstration district's voting-age population.

[16] We define the African-American population as all persons who identified themselves on the decennial census as Black alone or in one or more combinations with other race, and use a standard 50% threshold for determining a majority minority population district. The competition scoring criteria used a threshold of 48%. To accommodate these differences we include plans meeting the 48% threshold in the analysis, but our figures and tables continue to use the more standard 50% threshold.

scoring formula, the resulting number of county fragments is subtracted from fifty to arrive at an overall score for county splits.[17]

There are many ways to calculate district compactness (Niemi, et al. 1990). We employ what is commonly called the Polsby-Popper measure (invented by Cox 1927), which is ratio of a district's perimeter to a circle's perimeter with the same area as the district. Each district is scored on a scale of zero to one, with one being more compact and the overall plan score is the average across all districts. Competition organizers scored plans using the Roeck (1961) measure, which is the ratio of the area of a district to its minimally-sized bounding circle, with the idea that a circle is the most compact shape. The Roeck measure similarly scores districts between zero and one, with one being the most compact district. The resulting ratio is then averaged across all districts, rounded to a tenth of one percent, and multiplied by a hundred for incorporation into the competition formula.

The number of competitive districts and the representational fairness scores are generally calculated from the same underlying measure of districts' partisanship. We use the 2008 presidential vote, as two-party presidential vote is a measure of district partisanship used commonly in scholarly studies (e.g., Glazer, Grofman and Robbins 1987; Gelman and King 1994; Chen and Rodden 2013). Scholars tend to analyze presidential elections since they are the only national election and thus provide a common metric across states. The competition organizers calculated a "partisan index," the average of the two-party candidates' votes in four competitive statewide races: 2008 President, 2010 Governor, 2010 Auditor, and 2010 Secretary of State. For both metrics, the elections were closely contested, reflecting the overall competitiveness of Ohio.

[17] When scoring county splits consistent with the competition scoring, if a plan was submitted for scoring and has a county splits score, we use that score. If it was not submitted for scoring, we calculate a score. These latter plans may not have exceptions that would improve their score.

The statewide 2008 two-party presidential vote is 52.3% while the average of the four statewide elections is 48.0%, a difference that affects the analysis as we describe below.

We use a simple statistic to score competitive districts: the number of districts with a two-party 2008 presidential vote within a [.45, .55] range. This range is arbitrary, but has foundation in prior research (Swain, Borrelli and Reed 1998; McDonald 2006b). Competition organizers used a similar approach, but divided districts into four categories that give more weight to districts with tighter competitive margins. Three points are awarded for each 'heavily competitive' district, with a partisan index percentage within a [47.5, 52.5] range. Two points are awarded for each 'generally competitive' district within a [45.0, 47.5] or [52.5, 55.0] range. One point is awarded for each 'generally non-competitive' district in a [42.5, 45.0] or [55.0, 57.5] range. Zero points are awarded for all 'heavily non-competitive' districts outside these ranges. The overall competitiveness of a plan is computed by summing these scores across all districts.

We use a slightly modified version of the common scholarly metric to assess the partisan fairness of a plan, known as partisan bias (e.g., Tufte 1973, Stokes 1993). Scholars typically measure partisan bias as the vote share needed to win fifty percent of the seats, as the deviation from perfect proportionality of fifty percent of votes winning fifty percent of seats. We simply count the number of districts where the partisanship measure -- shifted to simulate a hypothetical fifty-fifty election -- is above or below fifty percent.[18] Competition organizers used a measure of partisan fairness that combines the concepts of partisan bias and a related concept known as responsiveness. Scholars (Tufte 1973) commonly conceive responsiveness as the slope of the expected seats to votes curve. In the neighborhood around fifty percent of the vote, responsiveness measures competitive districts since the slope is steeper if there are more competitive districts. The

[18] The partisan fairness measures are unlike the competition computations, in that the competition measures are not normalized to reflect the actual partisanship of the state, not a hypothetical fifty-fifty election.

competition organizers first computed the partisan balance of a plan by classifying districts into five categories. A strong Republican district has a partisan index of 45% or less. A Republican-leaning district is in a [45.0, 49.0] range. An even district is in a [49.0, 51.0] range. A Democratic-leaning district is in a [51.0, 55.0] range. A heavily Democratic district has a value of 55% or higher. A normalization to a perfectly competitive election and a complex weighting scheme is then applied.[19]

The competition organizers computed an overall score for each plan submitted for judging, which was the simple sum of the individual scores for county splits, compactness, competitiveness, and partisan fairness. The software was configured such that when a plan was submitted for scoring, if a plan met the threshold criteria the overall scores would then be calculated and posted on a leaderboard ranking the best scoring plans.

Analysis

We analyze plans on two sets of metrics, as described above. One set of metrics is consistent with prior analyses for Virginia and Florida (Altman and McDonald 2013, 2014) and comprises compactness, minority representation, county integrity, partisan balance, and competitiveness. The other set of metrics consists of the competition score and its four components, which we had no role in devising. In examining the competition's formula, we are interested in how formula components relate to the overall score to determine if individual components have more or less variation. We analyze the adopted plan, two bills created by the

[19] A denominator for a ratio measure is calculated by summing the strong Democratic and Republican districts, multiplied by 1.5, and adding to this the sum of the number of lean Democratic and Republican districts and even districts. A numerator is then calculated by summing the strong Democratic and Republican districts, multiplied by 1.5, adding to this the number of lean Democratic and Republican districts, and adding to this the number of even districts multiplied by two. The overall representational fairness score of a plan is computed by multiplying the ratio by 100 and rounding to a tenth of one percent. Next, an electoral disproportionality score is calculated by subtracting the statewide partisan index (51.4%) from the partisan balance score. For inclusion in the weighted overall competition score, the disproportionality score is subtracted from twenty-five and multiplied by four.

legislature, and those authored by the public. We include the 68 public plans with 16 districts that meet minimum legal requirements of assigning all geography and are within a one percent population deviation between the largest and smallest population district. Although only plans that had one majority African-American district were accepted for judging and all plans considered by the legislature had one such district, we include all publicly-shared plans in our analysis that meet our requirements, including those with no majority-minority district as this provides potential insights into trade-offs between minority representation and other criteria.

[Tables 1 & 2 Here]

We begin our analysis with the measures we have calculated for other states (Altman and McDonald 2013, 2014). The summary statistics in Table 1 reveal how the adopted plan, the average legislative plan, and average public plan differ overall on these measures (the overall competition score is also included for comparison). Table 2 compares the plan based on the competition measures. A number of features are notable.

First, the public plans score quite well. On average they score better than the adopted plan on compactness, county integrity, partisan balance, and district competitiveness, as well as on the composite competition score. All of these differences are statistically significant at the .95 level.[20] Public plans are negligibly worse on average -- but not statistically significantly different -- from the legislative plans on population equality. Many of the public plans have a population deviation of zero, which is an impressive technical feat for non-experts. Second, the public plans show a greater range as a whole -- demonstrating more possibilities. Third, A substantial number of public

[20] Calculations of statistical significance assume that the plans are independent draws from the same population of feasible redistricting plans. Since this assumption is questionable, any interpretation of significance should be taken with caution—nevertheless, it is clear that the difference between the "average" public plan and the legislative plan is both systematic (nonrandom) and substantively important.

plans failed to create a majority-minority district. However, excluding these plans does not change these observations.

[Figures 1 & 2 Here]

Simple descriptive statistics naturally mask characteristics of individual plans. So that individual plans may be visually compared against one another, we present scatter plots of individual plan statistics in Figures 1 and 2. Figure 1 shows the components of the score in the competition and Figure 2 displays the measures we have calculated for other states, along with the total competition score. In these plots, the public plans are identified with a triangle, the legislative plans with a circle, we label the adopted plan with an "A", and we label the winning competition plan with a "W". We draw clouds (99% density ellipses) around the public plans so that they may generally be distinguished from one another. There are only three legislative plans, often with the same value, so we cannot draw density ellipses around these plans.

Figure 1 and Figure 2 offer visual confirmation of the descriptive statistics that the public plans cover a larger "possibility space" than the legislature's plans. This is trivial since there are only three legislative plans. More interesting is that not only does a public plan dominate the legislative plans on every criterion, a public plan exists that dominates the legislature's plans for every pairwise combination of criteria. In other words, for any pair of criteria, there is always a publicly submitted plan that scores better than the legislative plan on both criteria.

Figure 1 shows the winning plan is consistently among the highest scoring plans on all components of the overall competition score, while the legislature's adopted plan consistently scores the lowest. Thus, the reform advocates can get much more of what *they* value over the legislative plan, without needing to sacrifice one criterion for another. The winning plan also has one African-American district (at 48% Black voting-age population). Such a district would likely

be performant for the minority candidate of choice,[21] and thus reformers' goals can be achieved harmoniously with the voting rights community's goals.

Figure 2 demonstrates that the winning competition plan does not score as highly when we evaluate it on alternative implementations of substantively similar criteria, particularly on compactness, partisan fairness, and district competition. The adopted plan continues to score lowly on the alternative implementations, except on district competition. These scoring changes suggest that reform goals cannot be fully disentangled from implementation details.

A comparison of our competitiveness measure and the competition organizers' measure illuminates how measurement choices matter. Recall that our measure of district partisanship is the 2008 presidential vote, which is about 4.2 percentage points more Democratic than the competition organizers' average of four statewide elections. The Republican's optimal strategy apparently resulted in the creation six districts with an electorally efficient level of partisan strength (Cain 1984) at just above 45% Democratic performance using our measure. In contrast, the adopted plan scores poorly on the organizers' measure. When using the organizers' measure these six districts are classified as being within the range [.425, .45] or lower, where districts receive only one point each towards a plan's competitiveness score. The competition's winning plan has ten districts between 50% and 52.5% Democratic performance using the organizers' measure, rewarding the plan with three points each towards the plan's competitiveness score. Five of these districts are classified above 55% Democratic performance by our measure, which is outside our classification of a competitive district. Thus, the strategies employed by the legislature and the winning plan

[21] Our impression is informed by racial bloc voting analyses conducted in other states, we have not conducted an Ohio analysis.

author to meet different target levels of partisan strength yield dramatically different scores, essentially flipping the ranking of the adopted plan and the winning competition plan.[22]

[Figure 3 Here]

We further explore the tradeoffs among three substantive, politically-relevant outcomes in Figure 3, using our evaluation measures: partisan balance, district competition, and the number of majority-minority districts. Figure 2 highlights the *Pareto frontier* across this set of three criteria. The Pareto frontier is the set of submitted plans that represent efficient trade-offs between criteria. Any plan not on the Pareto frontier can be beaten—or dominated—on *all* of the substantive criteria by plans on the frontier. In the plots on the left of Figure 3, heavy blue lines illustrate the frontier among pairs of criteria. Highlighted points show plans that are undominated on all three criteria simultaneously. A caution, as mentioned above, because redistricting is such a complex partitioning problem we cannot be certain that a plan does not exist that does better than the observed Pareto frontier. We observe that the Pareto frontier is quite flat – this indicates that in Ohio, not much trade-off is required across these criteria, one can do relatively well on all three criteria at once. Notably, the adopted plan is not on the Pareto frontier -- public plans exist that do better on all three criteria.

[Figure 4 and 5]

In Figures 4 and 5 we plot pairwise Pareto frontiers of the political outcomes of partisan fairness and competition, using our evaluation measures plus the competitions' overall score. We again draw lines to represent the Pareto frontier, with the addition that we draw circles around single or (apparently) disconnected points on the frontier. We unfold the partisan balance measure so that it now represents the number of Democratic leaning seats relative to an even partisan

[22] Judging which is measure is the most valid is outside the scope of this manuscript, but deserves further investigation.

division of the sixteen districts. Figure 4 provides insights as to why the legislature chose the adopted plan over potential alternative plans. The adopted plan is almost always on the left-most edge of the Pareto-frontier of the graphs in Figure 4. This suggests that the adopted plan was drawn to maximize the expected Republican seat advantage, secondary to other criteria. Interestingly, the tradeoff between Democratic seats and the overall competition score at bottom of Figure 4 reveals that although the score is presented as non-partisan, in practice, it appears to favor the Democrats in that higher scoring plans tended to have a greater number of Democratic-majority districts.

In Figure 5 we plot our criteria, including the unfolded partisan balance measure, and the overall competition score against our measure of district competitiveness. Per our above discussion, the adopted plan scores well on our measure of district competitiveness. Note, however, that the adopted plan is never on the Pareto frontier. This is true even with respect to partisan advantage: plans exist that could have given Republicans the same degree of partisan advantage with the addition of a competitive district. This evidence suggests that the adopted plan was not drawn to maximize district competitiveness, either.

[Figure 6]

In Figure 6 we compare the tradeoff of Democratic seat advantage with each component of the competition's score, along with regression lines and statistical margins of error. The overall regression line and Pareto frontier of all components of the competition score are strongly associated with increasing Democratic seat advantage. Was the competition "rigged" for Democrats? We lack determinative evidence: the competition rules could have been intentionally or unintentionally weighted toward Democratic seats (a second-order bias effect); or simply attracted more Democratic leaning contestants. (However, Illinois Republican State Representative Mike Fortner is the author of the winning plan.) Both of these explanations might yield similar

patterns. Suggestive that second-order bias is present in the scoring criteria, we note that although both the competition's measures of district competitiveness and partisan fairness are associated with increasing Democratic seat advantage, our *previous* measures of these criteria, used in other studies and displayed in Figures 4 and 5, did not show this association.

Discussion

The public's creation of large numbers of redistricting plans was catalyzed by information technology. The results here reinforce findings from analyses of public redistricting in a Virginia public competition and Florida public participation in which the public also created large numbers of redistricting plans for the first time in each state (Altman and McDonald 2013, 2014). These plans illuminate the range of redistricting outcomes that are possible, the motives of the legislature, and the interaction of criteria, and help us evaluate the performance of the redistricting by formula approach.

We are cautiously optimistic that redistricting by formula in Ohio can produce a better outcome than the congressional plan adopted by the legislature, in terms of those criteria that are typically regarded as politically-neutral and in terms of political goals such as partisan fairness. The latter is interesting only insofar that the methods we use here validate assertions that Ohio's congressional districts greatly favor the Republicans when compared to the state's relatively balanced partisanship (e.g., Wang 2016). Ohio reformers goals can be achieved without detriment to minority representation, too, which means the goals of the reform and voting rights advocacy communities can be in harmony.

Our conclusion with respect to district competitiveness is ambiguous, and underscores some of the challenges present in redistricting by formula. We arrive at an indeterminate evaluation of district competition because the scoring of plans depends on which elections are

chosen and how districts' levels of competition are scored. Similarly, plans scored differently and changed their rank ordering on compactness when evaluated on the Polsby-Popper measure compared to the Reock measure. While rank ordering of plans is typically similar under both measures, these compactness measures use different scales --- which can affect overall composite scores that are a function of individual weights (Altman 1998). Thus, seemingly-inconsequential details matter to evaluation of redistricting plans, such as which compactness measure to use, which elections will be used to to evaluate political outcomes, how to score plans based on the chosen political data matter to the rank-ordering of the evaluation of plans.

There may be further subtle biases in how criteria interact. For example, the winning plan performs well on partisan fairness by creating more districts that slightly lean towards the Democrats, and we note an apparent relationship between higher scoring plans on the competition's evaluation formula and Democratic advantage. We suspect such second-order bias may arise from the interaction of the threshold requirement for a minority district with district competition. Niemi and Deegan (1978) demonstrate formally that maximizing district competitiveness benefits a state's minority party. The creation of a heavily-Democratic African-American district carves out a Republican-leaning remainder of the state, such that when a district competitiveness requirement is imposed on the remainder, Democrats may benefit. This insight suggests if fairness and district competition are valued the dynamic between minority voting-rights and district competition should be considered.

The results of the competition's redistricting by formula approach suggest that a citizen participation can improve Ohio's redistricting. Allowing many members of the public to participate enables a wider exploration of the trade-offs among redistricting criteria than the legislature. The public is often able to do better on the criteria of compactness, respect for political boundaries,

district competition, and partisan fairness. Furthermore, the winning plan has a high Black voting-age population district, demonstrating that reformer goals can be achieved without expense to minority voting rights.

Despite the success of the quality of the plans produced by the competition, public participation in the process appears to have little influence on the legislature's plan. This reinforces the need to design commissions that are independent. As we have argued elsewhere, effective redistricting commissions should accept and favorably weigh public input, should not be subject to legislative veto or modification, and should have permanent funding and staffing free from legislative manipulation (Altman and McDonald 2014b).

These insights are relevant, in part, to Ohio's newly reformed state legislative commission, which, if reformers have their way, will be extended to congressional redistricting. The commission is not "independent" in the sense that the commission's members are partisan appointees. However, the amended Ohio constitution now has an explicit requirement that "statewide proportion of districts...shall correspond closely to the statewide preferences of the voters of Ohio" (Ohio Constitution Article XI § 3). We strongly recommend that the evaluation measure for this and other constitutional requirements be established in a transparent manner with bipartisan agreement prior to the release of census population data, so that opportunities for *post-hoc* manipulation are minimized. Since there are a number of other geographical criteria in the constitution, and we observe the having more eyes on a problem can reveal a wider range of viable solutions, we also recommend that the commission solicit public input.

We conclude that commissions should not be forced to follow automatic quantitative criteria. Instead, they should be authorized to make fair judgments using all socially and politically relevant information subject to complete operational transparency. However, we are cognizant that

when given discretion, redistricting authorities often use the opportunity to gerrymander and the courts will often be deferential. The public can generate legal plans that score highly on a formula, and the public tends to approach redistricting in a fundamentally different way than politicians. These public can provide informative counterfactual plans to assist in inferring legislative intent. We thus believe that redistricting by formula is a plausible approach to reforming redistricting and encourage continued experimentation.

		Legislature	Public
Number of Plans		3	63
Compactness	*Mean*	0.114	0.161
	Std Err	0.002	0.002
	Max	0.119	0.187
	Range	0.007	0.079
Majority-Minority	*Mean*	1.000	0.270
	Std Err	0.000	0.056
	Max	1.000	1.000
	Range	0.000	1.000
Population Equality	*Mean*	0.00	0.01
	Std Err	0.000	0.000
	Max	0.00	0.00
	Range	0.000	0.010
Partisan Balance	*Mean*	-4.00	-0.97
	Std Err	0.000	0.111
	Max	-4.00	0.000
	Range	0.000	4.000
County integrity	*Mean*	-55.0	-29.3
	Std Err	0.000	1.995
	Max	-55.0	-11.0
	Range	0.000	85.00
Competitiveness	*Mean*	9.000	6.349
	Std Err	0.000	0.209

	Max	9.000	10.00
	Range	0.000	7.000
Total Score - OHC	*Mean*	53.63	159.9
	Std Err	0.333	4.481
	Max	54.30	222.6
	Range	1.000	184.7

Table 1: Congressional Plan Statistics.

Notes: Legislature and Public categories summarize all legislative proposals and public submissions meeting threshold requirements as described in text. OHC is "Ohio Competition".

		Legislature	**Public**
Number of Plans		3	63
Compactness - OHC	*Mean*	34.33	42.86
	Std Err	0.333	0.431
	Max	35.00	49.00
	Range	1.000	15.80
County Splits - OHC	*Mean*	-4.00	24.11
	Std Err	0.000	2.086
	Max	-4.00	43.00
	Range	0.000	88.00
Fairness - OHC	*Mean*	12.30	69.01
	Std Err	0.000	2.935
	Max	12.30	105.6
	Range	0.000	100.0
Competitiveness - OHC	*Mean*	11.00	23.95
	Std Err	0.000	0.553

	Max	11.00	33.00
	Range	0.000	23.00
Total Score - OHC	*Mean*	53.63	159.9
	Std Err	0.333	4.481
	Max	54.30	222.6
	Range	1.000	184.7

Table 2: Congressional Plan Ohio Competition Scores.

Notes: Legislature and Public categories summarize all legislative proposals and public submissions meeting threshold requirements as described in text. OHC is "Ohio Competition".

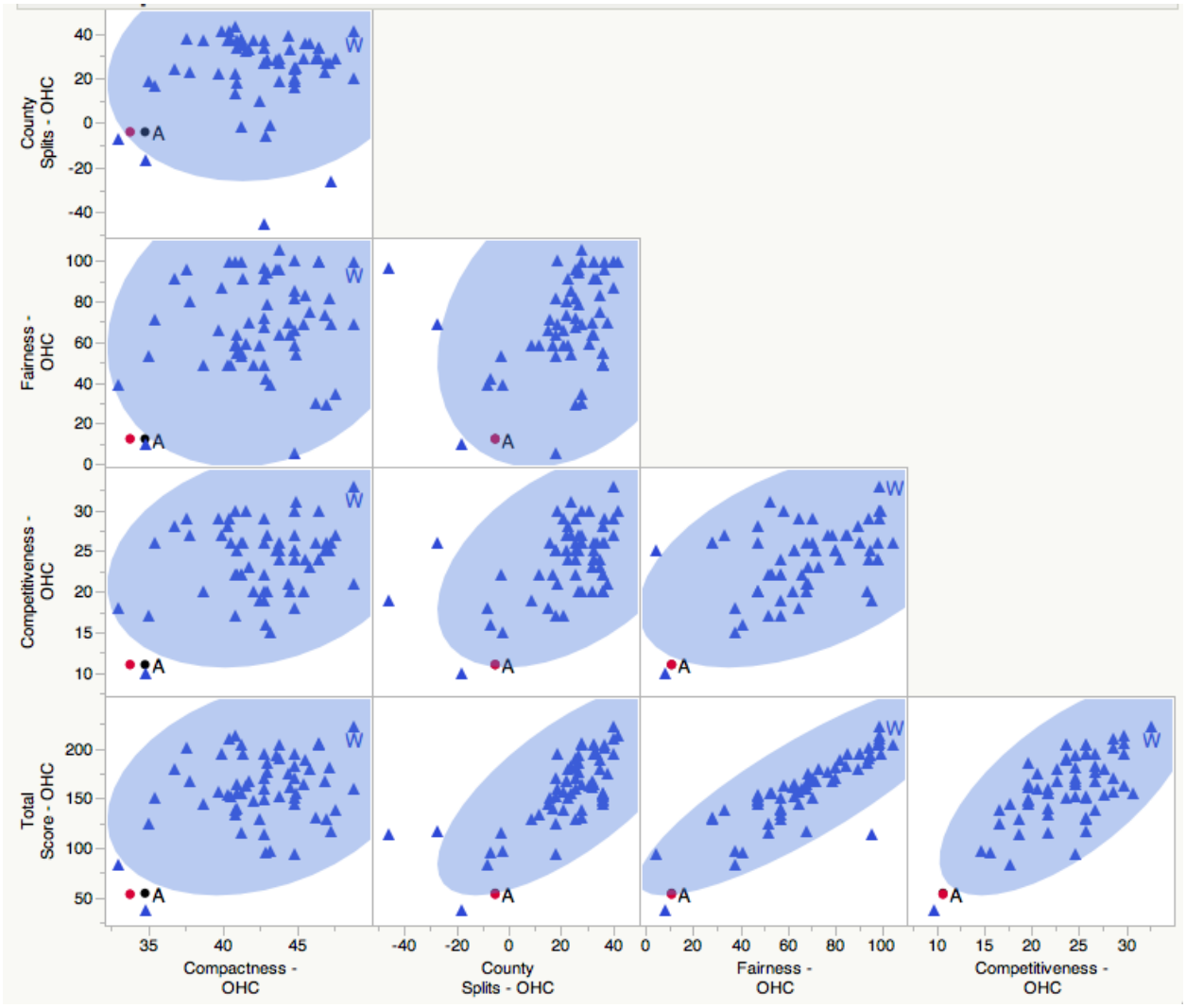

Figure 1: Pairwise Congressional Score Comparisons (Scatterplots) - Ohio Competition Scores

Notes: Legislature plans identified as circles. Public plans identified as triangles. Adopted plan labeled as "A". Winning competition plan labeled as "W". Clouds (99% density ellipses) drawn around public plans; not practical to draw similar ellipses for the three legislative plans. For ease of comparison, scores in these scatter plots have been transformed so that an increase on either axis reflects a substantively better score. Please see text for definition of measures.

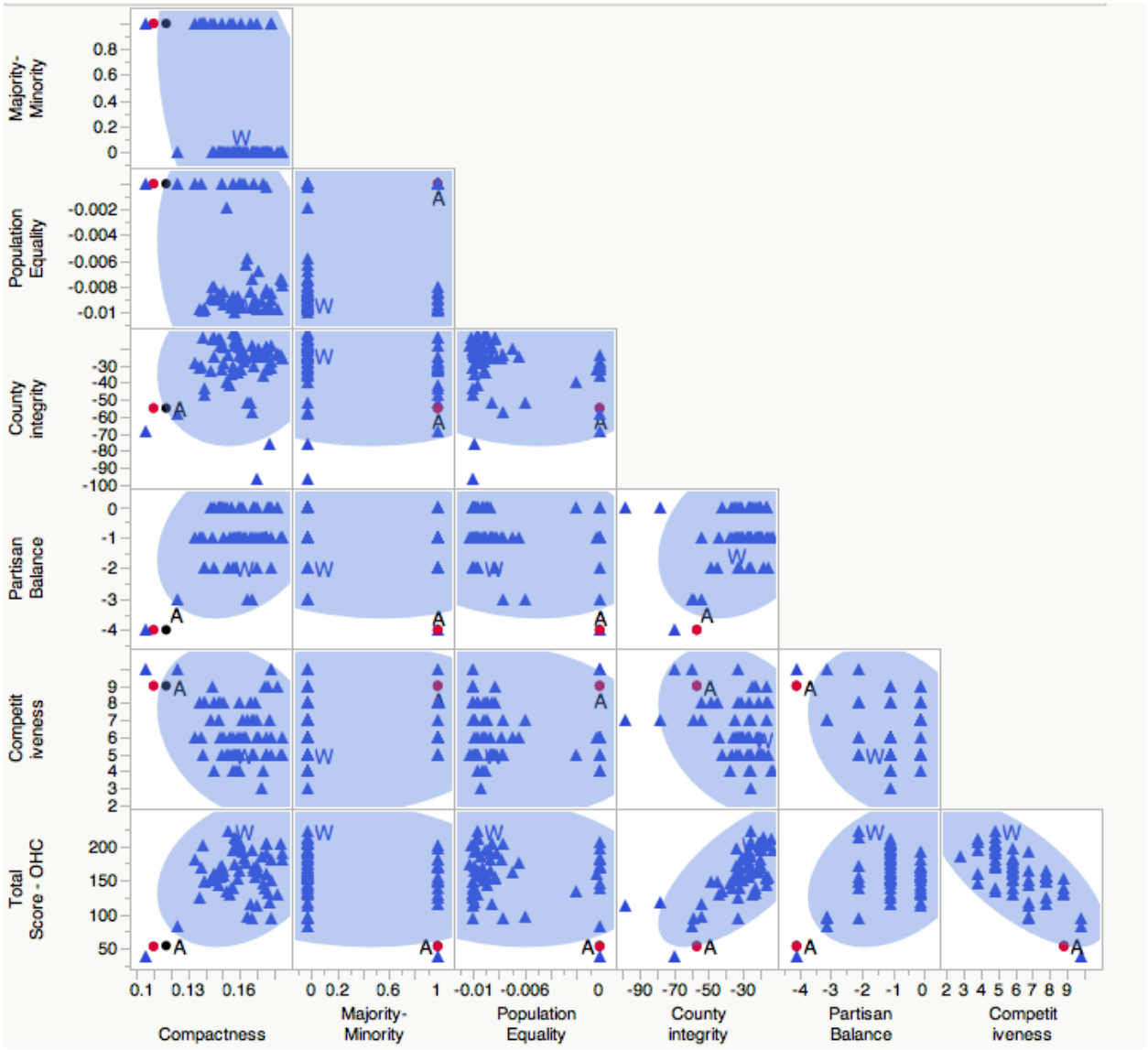

Figure 2: Pairwise Congressional Score Comparisons (Scatterplots) - Standardized Scores

Notes: Legislature plans identified as circles. Public plans identified as triangles. Adopted plan labeled as "A". Winning competition plan labeled as "W". Clouds (99% density ellipses) drawn around public plans; not practical to draw similar ellipses for the three legislative plans. For ease of comparison, scores in these scatter plots have been transformed so that an increase on either axis reflects a substantively better score. Thus, population equality is plotted as -1 times the absolute deviation from 0 (0 is the best score); county integrity is plotted as -1 times the number of county splits (0 is the best score); and partisan balance is plotted as -1 times the absolute deviation from a 50/50 seat split (0 is the best score). Please see text for definition of measures.

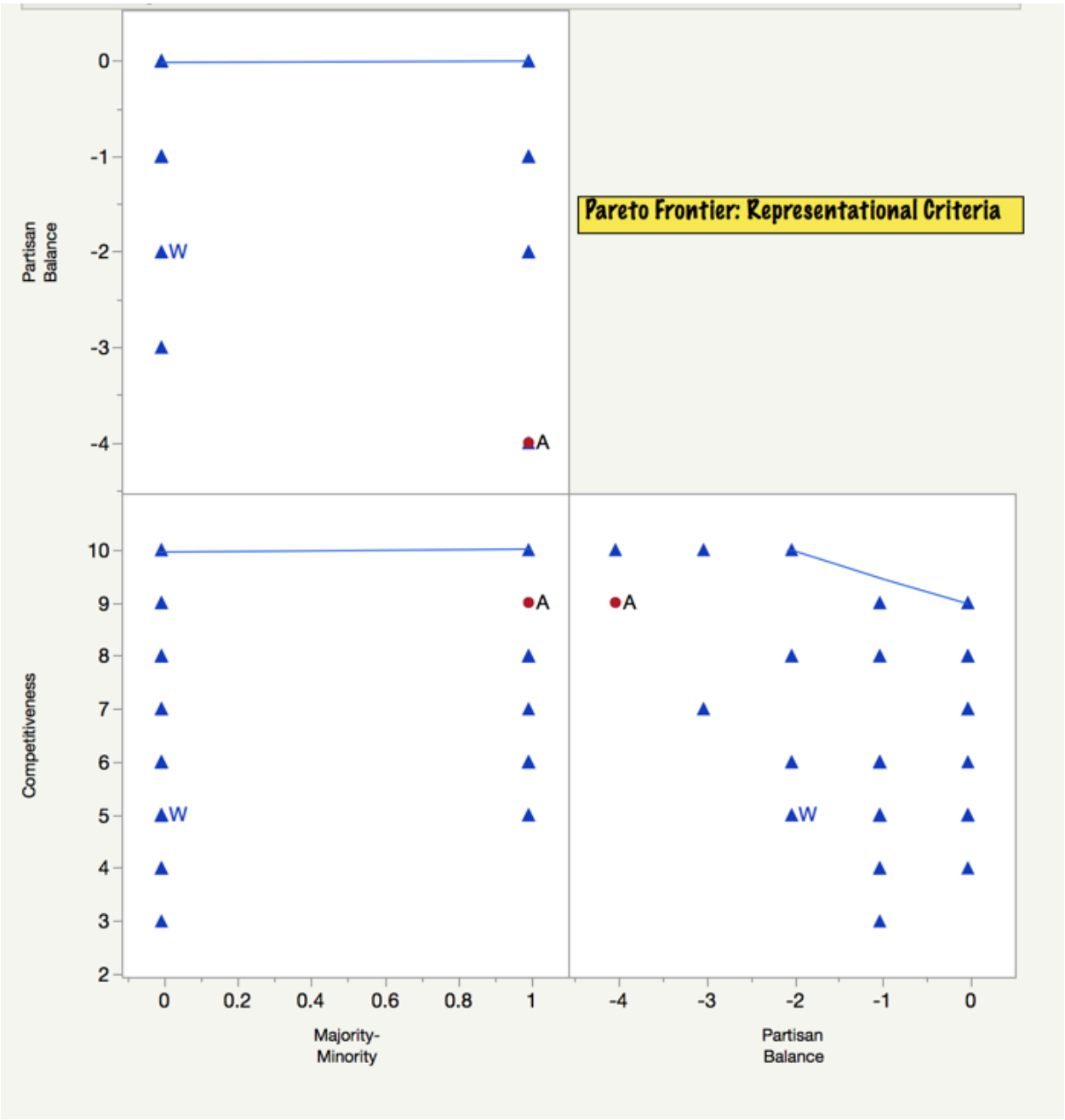

Figure 3: Pareto frontier across the three political and substantively varying criteria.

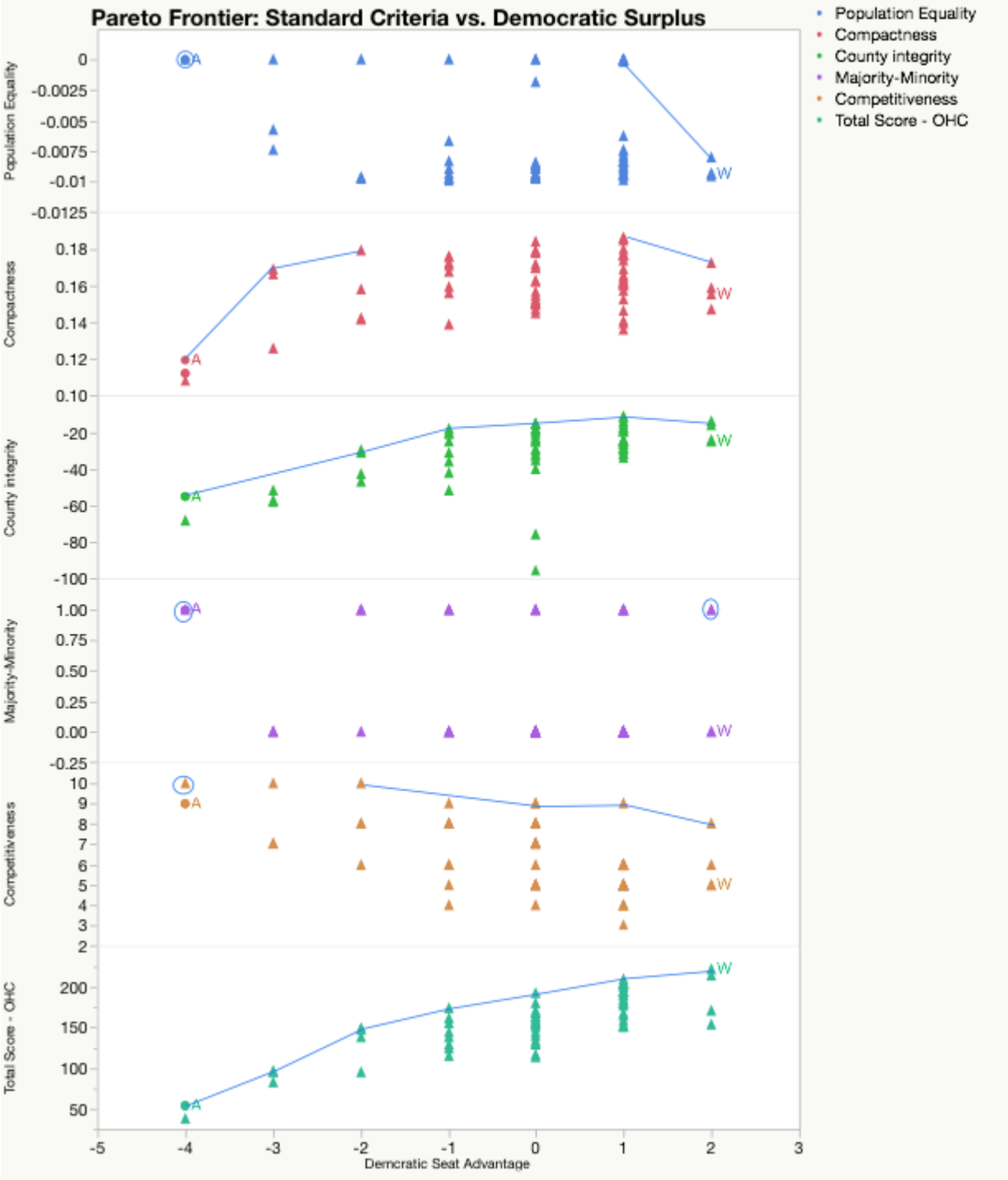

Figure 4: Democratic Seat Advantage vs. Standard Criteria

Notes: Heavy colored lines illustrate the frontier among pairs of criteria.

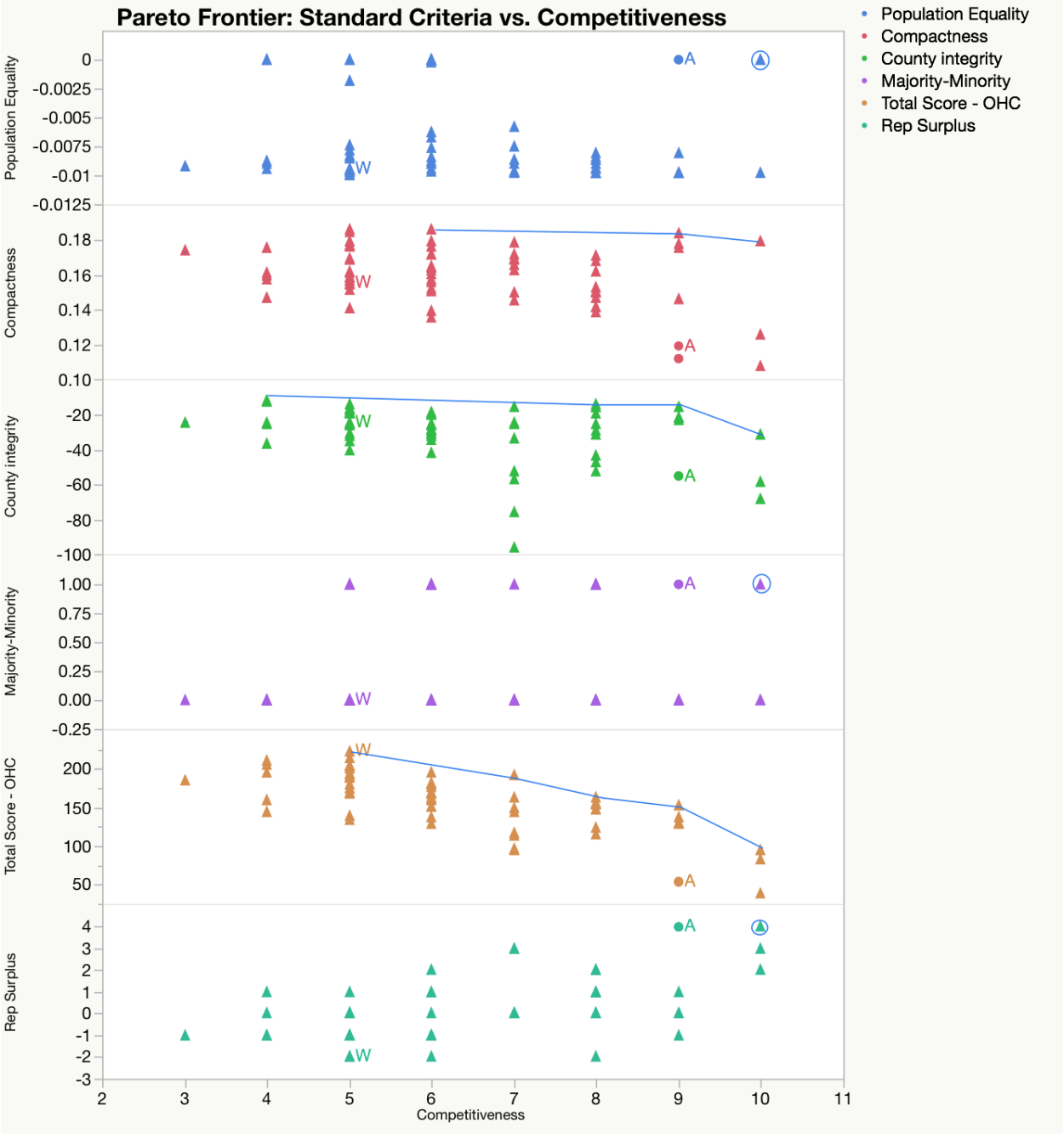

Figure 5: Competitiveness vs. Standard Criteria

Notes: Heavy colored lines illustrate the frontier among pairs of criteria.

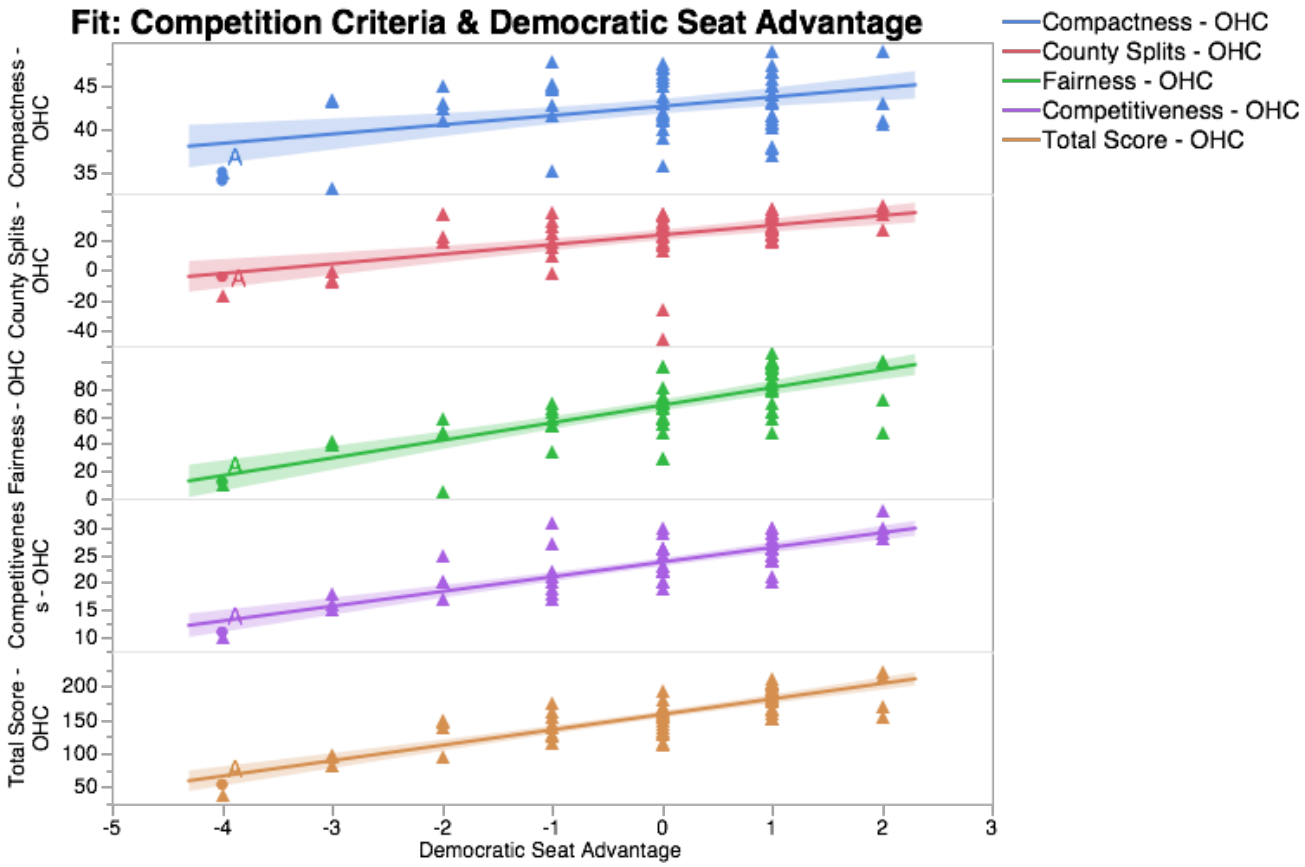

Figure 6: Democratic Seat Advantage vs. Standard Criteria.

Notes: Lines and shaded areas indicate the regression line between the individual score component and the democratic seat advantage.

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